The Complete Basis Set of The Orthonormal Vector Polynomials in A unit Annular Circular Pupil

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Received Date: 1 / 11 / 2015
Accepted Date: 24 / 6 / 2016

الخلاصة

تم في هذا البحث اشتقاق المجموعة الكاملة لمتعددات الحدود المتجهة العيارية والمتعامدة لفتحة حلقية مساحتها الخارجية تساوي وحدة واحدة بإيجاد أولا مجموعة الدوال المتعامدة والتي تمثل كميات متجهة يمكن الحصول عليها بأخذ انحدار دوال زرنيك الحلقية PZ ، ويمكن الحصول على التعامد باستخدام طريقة كرام شمت للتعامد. العلاقة بين هذه المتعددات ومتعددات حدود زرنيك للفتحة الدائرية وانحدارات متعددات حدود زرنيك الدائرية تم ايجادها ايضا في هذا البحث.

بعد ذلك، ولإكمال المجموعة، تم اضافة مجموعة متممة من الدوال والتي لها انحدار يساوي صفر، وهي ما يطلق عليها بالالتفاف أو الدوران.

الكلمات المفتاحية

متعددات الحدود المتجهة العيارية، انحدار دوال زرنيك الحلقية zp، طريقة التعامد ل (كرام شمدت).

Abstract

In this paper, a complete set of the orthonormal vector polynomials in a unit annular pupil were derived by finding, first a set of orthogonal functions that represent vector quantities which can be generated from the gradients of annular Zernike polynomials ZP, and the orthogonality is made by MATLAB code using Gram Schmidt orthogonalization method. A relation of these polynomials with the circular ZP and circular ZP gradients are represented also in this work.

Then, to complete the basis, a complementary set of functions were added that have zero divergence, those which are defined locally as a rotation or curl.

Keywords

Orthonormal vector polynomials, Annular Zernike polynomials ZP, Gram Schmidt orthogonalization method .

Vol. 4, No. 7 and 8 P. (67-80)E, 2016
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1. Introduction

Many researchers were having an interest in annular pupils for their use in optical systems and several of them were studied them with Zernike polynomials [1-11].

The vector Zernike polynomials or Zernike gradients are also important to study. Derivatives of Zernike polynomials can be useful whenever the gradient of a wavefront is required. Wavefront gradients occur in some geometrical optics problems as well as direct measurements in an electronic Hartmann Test. These vector functions have immediate application for fitting data from a Shack-Hartmann wavefront sensor or for fitting mapping distortion for optical testing.

These polynomials are studied for circular aperture in (1976) by Robert J. Noll, who gave the rules for computing the derivatives of Zernike polynomials as a linear combination of the polynomials themselves. [12].

In (2007), Zhao and Burge provide a set of complete basis for representing vector fields that can be defined as a gradient of some scalar functions across circular pupil. These polynomials can be transformed to the scalar circular ZPs [13]. Then in the next year, in (2008), they gave an additional set of vector functions consisting only of rotational terms with zero divergence [14]. These two sets together provide a complete basis that can represent all vector distributions in a circular domain. In (2009), they justified, with examples, why the set of vector polynomials is the appropriate choice for describing mapping distortions, and they showed the excellent fitting results with the polynomials [15].

In this work, the first (21) annular ZPs were found using the first (21) circular ZP and transform them to an orthonormal polynomials using Gram Schmidt orthogonalization method (GSOM) (Which transform non orthogonal polynomials to orthogonal ones) and the normality law (which transform non normal polynomials to normal ones). Then a set of vector polynomials are presented, which are orthonormal in a unit annular aperture with obscuration ratio equal to (ϵ). These polynomials are perfect for fitting slope data, and the fitted slope map can be easily converted to the wavefront map expressed in terms of Zernike polynomials.

But since these polynomials are gradients of linear combinations of ZPs, they have zero curl, which means they make an incomplete set of vector polynomials, such that an arbitrary continuously differentiable vector function defined over a unit annular pupil cannot be represented by linear combinations of these polynomials. So, additional vector polynomials were derived and added to make a complete set of vector polynomials.

In the next section, a procedure for deriving the orthonormal annular ZP using (GSOM) and the circular Zernike polynomials were produced. Then the gradients of the annular ZP are calculated, and by using, for the second time, the (GSOM) the orthonormal vector annular ZPs were found. Then the relation of them with both the circular ZPs and circular gradient ZPs have been found, and several equations representing these relations were concluded. And in the end of this section, a complementary set of vector polynomials were derived to have a complete set of vector polynomials. Finally, a discussion were made for the results in section (3).

2. Results

2.1. Annular circular Zernike polynomials

There are different numbering schemes for circular ZPs, and in this work, Noll's notation has been adopted which is the same as what considered by C. Zhao and J. Burge [13].

Circular ZPs are not suitable for annular pupils. So, these polynomials must be converted to annular ZPs, and this could be done with (GSOM) [16], which can be illustrated by the following equation:

$$A_2'(\rho,\theta;\epsilon) = A_2(\rho,\theta;\epsilon) - \frac{\int_0^{2\pi}\int_0^1 A_1(\rho,\theta;\epsilon)A_2(\rho,\theta;\epsilon)\,\rho d\rho d\theta - \int_0^{2\pi}\int_0^{\epsilon} A_1(\rho,\theta;\epsilon)A_2(\rho,\theta;\epsilon)\,\rho d\rho d\theta}{\int_0^{2\pi}\int_0^1 A_1^2(\rho,\theta;\epsilon)\rho d\rho d\theta - \int_0^{2\pi}\int_0^{\epsilon} A_1^2(\rho,\theta;\epsilon)\rho d\rho d\theta} * A_1(\rho,\theta;\epsilon) \quad (1)$$

Where (A_1) and (A_2) are two non-orthogonal functions, while (A_2') is orthogonal with (A_1).

And to normalize these polynomials, the normalization rule must be submitted:

$$C^2 \frac{\int_0^{2\pi}\int_0^1 A_1^2(\rho,\theta;\epsilon)\rho\, d\rho d\theta - \int_0^{2\pi}\int_0^{\epsilon} A_1^2(\rho,\theta;\epsilon)\rho d\rho\, d\theta}{\int_0^{2\pi}\int_0^1 \rho d\rho\, d\theta - \int_0^{2\pi}\int_0^{\epsilon} \rho\, d\rho d\theta} = 1 \qquad (2)$$

The above two equations were programmed in MATLAB, and the results were illustrated in Table (1), which show the first twenty one orthonormal annular ZP in polar coordinates, for annular pupil with obscuration ratio(ϵ). These polynomials can be reduced to circular ZPs by putting (ϵ =0).

Table (2), shows the annular ZPs in Cartesian coordinates, the conversion is made also using a MATLAB code using these equations:

$x=r\cos(\theta)$, $y=r\sin(\theta)$, $r^2=x^2+y^2$, $\theta=\tan^{-1}(y/(x)$ (3)

Then the relationships between the annular ZP and the circular ZP were shown in Table (3), Where (Z_i) here represents circular ZPs. We can see from this table that annular ZP is a linear combination of at most three circular ZP, where when (n=m) it is proportional to one circular ZP, when (n-m=2), it is proportional to two circular ZP, while when (n-m=4), it is a linear combination of three circular ZP.

2.2. The Gradients of annular Zernike polynomials

To find the gradients of annular ZPs, a MATLAB code was written for this purpose. So, either the results in Table (2), which were written in Cartesian coordinates, are used to compute gradients with the equation:

$$\nabla A = \frac{\partial A}{\partial x}\hat{\imath} + \frac{\partial A}{\partial y}\hat{\jmath} \quad (4)$$

Or the results of Table (1) were used, which were written in polar coordinates,

$$\nabla A = \frac{\partial A}{\partial \rho}\hat{a}_\rho + \frac{\partial A}{\rho\,\partial\theta}\hat{a}_\theta \qquad (5)$$

Where ($\hat{a}_\rho$ and $\hat{a}_\theta$) represent the unit vectors in polar coordinates. Here a transformation of the coordinates to the Cartesian coordinates must be done to get the components of (∇A) in ($\hat{\imath}$ and $\hat{\jmath}$).

Table (4) represents the gradients of the first twenty one annular polynomials. These functions are not easy to work with, because they are not orthogonal to each other over an annular aperture. So to convert them to an orthonormal functions, equations (1) and (2) must be worked but for the annular gradient polynomials (vector polynomials).

The process now became sort of complicated, and the computer became slow in finding the functions. So this process can be done in another way by applying (GSOM) for gradients of Zernike circular aperture that can be computed from circular ZP [13], but over the annular aperture, i.e. the limits of integration will be the limits of annular.

The results of Table (5), show the orthonormal vector annular polynomials. Table (6), represents the orthonormal vector annular ZPs as function of circular ZPs, Z(x,y), while Table (7) represents them in terms of gradient of circular Zernike polynomials ∇Z(x,y).

It can be concluded from Table (7) that:

i) For all (j) with (n=m),

$$\vec{S}_j = \frac{1}{\sqrt{2n(n+1)(\epsilon^{2n-2}+\epsilon^{2n-4}+..+1)}}\nabla Z_j \qquad (6)$$

ii) And for all (j) with (n≠m), and (n-m=2)

$$\vec{S}_j = C_j\left[k\,\nabla Z_j - \sqrt{\frac{n+1}{n-1}}\,\nabla Z_{j'(n'=n-2,m'=m)}\left(n\epsilon^{2(n-2)} + \epsilon^{2(n-2)-2} + \cdots + 1\right)\right] \quad (7)$$

where (C) represent the normalization constant for the orthogonal vector annular ZP, Table (8), and $k=(\epsilon^{2(n-3)} + \epsilon^{2(n-3)-2} + \cdots +1)$ except for (n<3), (k=1).

iii) For (n≠m), (n-m=4)

$\vec{S}_j = C_j\,(k_1 \nabla Z_j - k_2\, \nabla Z_{j(n-2,m)} - k3\ \nabla Z_{j\,(n-4,m)})$ (8)

$k1=(2\epsilon^4-\epsilon^2+2)$

$$k_2=\sqrt{\frac{3}{2}}(10\epsilon^6 + 2\epsilon^4 - \epsilon^2 + 12),$$

$$k_3 = 10\sqrt{3}\,(\epsilon^8 + 3\epsilon^6 - 3\epsilon^4 + 2\epsilon^2 + 3)$$

where (j-j') is even when (m≠0).

In general, the vector polynomial ($\vec{S}$) is equal to:

$S_j = i\,S_{jx} + j\,S_{jy}$ (9)

Fig. (1) shows the plots of first (9) vector annular polynomials. The arrows represent the amount and direction of the displacement of a particular point.

2.3. The relation between the vector annular ZP and the scalar circular ZP

The space of vector distribution over the unit annular circular can be written as a linear combination of a set of ($\vec{S}$) polynomials:

$V = \sum_j \alpha_j S_j$ (10)

and it can be written as a gradient of scalar function:

$\vec{V} = \nabla\Phi$ (11)

but $\vec{S}_j = \nabla\phi_j$

so, from equations (6,7, and 8), we get:

i) For all (j) with (n=m),

$$\phi_j = \frac{1}{\sqrt{2n(n+1)(\epsilon^{2n-2}+\epsilon^{2n-4}+..+1)}} Z_j \quad (12)$$

ii) For all (j) with (n≠m), and (n-m=2)

$$\phi_j = C_j\left[k\,Z_j - \sqrt{\frac{n+1}{n-1}}\, Z_{j'(n'=n-2,m'=m)}\left(n\epsilon^{2(n-2)} + \epsilon^{2(n-2)-2} + \cdots + 1\right)\right] \quad (13)$$

iii) For (n≠m), (n-m=4)

$$\phi_j = C_j(k_1\, Z_j - k_2\, Z_{j(n-2,m)} - k_3\, Z_{j\,(n-4,m)}) \quad (14)$$

where (j-j') is even when (m≠0).

That means, the scalar polynomials (ϕ) can be found form the vector polynomials ($\vec{S}$), for example:

$$\vec{S}_7 = \left[\nabla Z_7 - \sqrt{2}(3\epsilon^2 + 1)\nabla Z_3\right]/\sqrt{48\epsilon^4 - 24\epsilon^2 + 48}$$

leads to:

$$\phi_7 = \left[Z_7 - \sqrt{2}(3\epsilon^2 + 1)Z_3\right]/\sqrt{48\epsilon^4 - 24\epsilon^2 + 48}$$

$$\vec{V} = \textstyle\sum_j \alpha_j S_j = \nabla\,\Phi \quad (15)$$

Then the scalar function can be written as a linear combination of standard circular ZP.

$$\Phi = \textstyle\sum_j \alpha_j \phi_j = \sum_j \gamma_j Z_j \quad (16)$$

where (γ_j) is:

i) For (j) with (n=m),

$$\gamma_j = \frac{\alpha_{j(n,m)}}{\sqrt{2n(n+1)(\epsilon^{2n-2}+\epsilon^{2n-4}+..+1)}} - \alpha_{j(\acute{n}=n+2,\acute{m}=m)}\, C_{j(n',m)}\sqrt{\frac{n'+1}{n'-1}}\left(n'\epsilon^{2(n'-2)} + \epsilon^{2(n'-2)-2} + ..+1\right) - \alpha_{j(\acute{n}=n+4,\acute{m}=m)} C_{j(n',m)} k_3 \quad (17)$$

ii) For all (j) with (n≠m) and (n-m=2),

$$\gamma_j = \alpha_{j(n,m)}\, C_{j(n,m)}\, k - \alpha_{j(\acute{n}=n+2,\acute{m}=m)} C_{j'(n'+m)}\sqrt{\frac{n'+1}{n'-1}}\left(n'\epsilon^{2(n'-2)} + \epsilon^{2(n'-2)-2} + \cdots + 1\right) \quad (18)$$

iii) For (j) with (n≠m) and (n-m=4),

$\gamma_j = C_j \alpha_{j(n,m)}$ (19)

2.4. Derivation of a complementary set of vector polynomials

Polynomials ($\vec{S}_2$) and ($\vec{S}_3$) are represent (x) and (y) translation respectively, and ($\vec{S}_4$) represents scaling. But no ($\vec{S}$) polynomial represents rotation. The reason is that the rotation vector has non-zero curl, while all ($\vec{S}$) polynomials have zero curl. So, we need a complementary set of vector polynomials which have zero divergence and non-zero curl. This new set combined with the zero-curl set ($\vec{S}$) to make a complete set such that it can represent any continuously differentiable vector polynomials defined over a unit annular pupil [14].

Any vector field can be written as [17]:

$$\vec{v} = \nabla\phi + \nabla \times \vec{P} \quad (20)$$

where (ϕ) is a scalar and ($\vec{P}$) is a vector. The divergence of ($\vec{v}$) is then:

$$\nabla \cdot \vec{v} = \nabla^2\phi + \nabla \cdot (\vec{\nabla} \times \vec{P}) = \nabla^2\phi \quad (21)$$

and the curl of ($\vec{v}$) is:

$$\vec{\nabla} \times \vec{v} = \nabla \times (\vec{\nabla} \times \vec{P}) = \nabla(\nabla \cdot \vec{P}) - \nabla^2\vec{P} \quad (22)$$

The ($\vec{S}$) polynomials presented in the previous sections, were defined as gradients of scalar functions, so have no curl component and ($\vec{P}$= 0). We complete the basis by adding a second set that has zero divergence, therefore zero (φ), but non zero ($\vec{P}$), such that: ($\vec{T} = \nabla\times\vec{P}$), this set has to be mutually orthogonal as well.

As illustrated by C. Zhao and J. Burge [14], Like the ($\vec{S}$) polynomials, ($\vec{T}$) polynomials are vectors defined in (x-y) plane only. A convenient choice of ($\vec{P}$) is vectors along (z) axis only, i.e. (Px=Py=0). We can use a scalar (ψ) instead to represent ($\vec{P}$):

$\vec{P} = \psi\hat{k}$ (23)

where (ψ) is a function of (x) and (y): (ψ = ψ (x, y)). It follows that:

$$\vec{T}_j = \nabla \times (\psi_j\hat{k}) = \frac{\partial\psi_J}{\partial y}\hat{\imath} - \frac{\partial\psi_J}{\partial x}\hat{\jmath} \quad (24)$$

The inner product of two ($\vec{T}$) polynomial must be:

$$(\vec{T}_i, \vec{T}_j) = \frac{1}{\pi(1-\epsilon^2)}\iint \left((\nabla\psi_i)\cdot(\nabla\psi_j)\right) dx\,dy = 1$$

for (i=j and 0) for (i≠j) (25)

A basis of functions {ψ_i} will be chosen to generate the ($\vec{T}$) polynomials that be the same basis as that used to generate the ($\vec{S}$) polynomials, {φ_i}. So, by letting (ψi=φi), we get Tables (9) and (10) are represent ($\vec{T}$) polynomials in Cartesian coordinates and in terms of circular Zernike polynomials respectively. Also the plots of first (9) ($\vec{T}$) polynomials are shown in fig. (2).

It can be seen that($\vec{T}_j$(x,y(and $\vec{S}_j$ (x,y)) have same magnitude and are orthogonal to each other at any point in a unit annular pupil, therefore ($\vec{T}_j \cdot \vec{S}_j$=0). But the sets ($\vec{S}$) and ($\vec{T}$) are not fully independent. For all (j) with (m=n). It can be shown that ($\vec{T}_j$) has (0) curl and is therefore not linearly independent of ($\vec{S}$) polynomials. For example, when (j=9 or 10), (m=n=3):

$$\vec{T}_9 = (Z_6\hat{\imath} - Z_5\hat{\jmath})/\sqrt{2} = \vec{S}_{10} \text{ and } \vec{T}_{10} = (-Z_5\hat{\imath} - Z_6\hat{\jmath})/\sqrt{2} = -\vec{S}_9.$$

3. Discussion:

By looking at the forms of annular ZPs, It can be seen that these polynomials have the same properties of that of circular ZP, they have axial symmetry (because they can be written in one form of triangular function (sin or cos), and circular symmetry (because they were separable in r and θ). When these polynomials were written in terms of circular ZPs, it can be concluded that they are a linear combination of at most three circular ZP, and when (n=m), Annular ZPs A(n,m) equal to the circular ZP, Z(n,m), multiplied by a constant depends on obscuration ratio (ϵ), while when (n-m=2), annular ZP is a linear combination of two circular ZP (Z(n,m) and Z(n-2,m), and when (n-m=4), A(n,m) is a linear combination of (Z(n,m) , Z(n-2), and Z(n-4,m)).

From table (6), which represent the ($\vec{S}$) as a function of (Z), the annular vector ZP is a linear combination of at most seven circular ZPs, unlike that of circular vector which contain at most four circular ZP, as in ref. 13.

As like as when the annular ZP with (ϵ=0) gives the circular ZP, the vector annular ZP are returned to circular vector polynomials when (ϵ=0), and the annular vector ZP, as shown in table (7), can be represented as linear combination of at most four circular ZP gradients, while the circular vector ZP is a linear combination of at most three circular ZP gradients, as in ref. 13. And again when (n=m) orthonormal annular vector ZPs equal to the circular ZP gradient multiplied by a constant depends on obscuration ratio (ϵ), while when

($n \neq m$) the ($\vec{S}$) is a linear combination of circular ZP gradient ($\nabla Z_{(n,m)}$, $\nabla Z_{(n-2)}$, and $\nabla Z_{(n-4)}$).

From table (5) of ($\vec{S}$), equations (6, 7, and 8) were concluded, which were used to conclude equations (12, 13, and 14) that illustrate the relation between the vector polynomials and the scalar circular ZP.

Because the ($\vec{S}$) polynomials are representing the divergence of a scalar functions (ϕ), it can be known from books of electromagnetism that the curl of these polynomials is equal to zero, so this makes these polynomials are not complete, and another work must be done to get the other polynomials with zero divergence and non-zero curl.

4. Conclusions

By looking to the vector fields ($\vec{S}$) and ($\vec{T}$), we can conclude that:

As ($\vec{S}$) have zero curl everywhere, then ($\vec{S}$) is known as irrotational vector fields.

Since ($\vec{S}$) functions are 2-D vectors defined in a plane, the curl can be expressed mathematically as line integral along a closed path in the plane [15]:

$$\int_s \nabla \times \vec{S} \cdot \vec{ds} = \oint_\ell \vec{S} \cdot \vec{dl} = 0$$

As ($\vec{T}$) have zero divergence everywhere, then ($\vec{T}$) is known as solenoidal vector fields.

As ($\vec{T}$) are also 2-D vectors defined in a plane, the divergence here can be expressed mathematically as a line integral over a closed path [17]:

$$\oint \vec{T} \cdot \hat{n}\, dl = 0$$

where ($\hat{n}$) is the unit normal vector pointing out of the closed path.

There is a region for two types of vector fields have both divergence and curls zero everywhere, this is known as Laplacian vector field.

$$\nabla^2 \phi_j = 0$$

References

[1] V. Mahajan, "Zernike annular polynomials for imaging systems with Annular pupils", J. Opt. Soc. Am., 71, 1, 75-85, 1981.

[2] V. Mahajan, ''Zernike Annular Polynomials and Optical Aberrations of Systems with Annular Pupils'', Optical Society of America, 5, 11, 8125-8127, 1994.

[3] L. Edward, ''Use of annular subapertures with focus control for testing rotationally symmetric optical systems'', Ph.D. Thesis,The University of Arizona, USA, 1987.

[4] G. Dai, ''Modal wave-front reconstruction with Zernike Polynomials and Karhunen–Lo`eve functions'', J. Opt. Soc. Am. A, 13, 6, 1218-1225, 1996.

[5] G. Dai, and V. Mahajan, ''Zernike annular polynomials and Atmospheric turbulence'', J. Opt. Soc. Am. A, 24, 1, 139-155, 2007.

[6] G. Dai, and V. Mahajan, ''Orthonormal polynomials in wavefront analysis: analytical solution'', J. Opt. Soc. Am. A, 24, 9, 2994-3016, 2007.

[7] G. Dai, and V. Mahajan, ''Orthonormal polynomials in wavefront analysis: error analysis'', OPTICS EXPRESS, 47, 19, 3433-3445, 2008.

[8] V. Mahajan, and M. Aftab, ''Systematic comparison of the use of annular and Zernike circle polynomials for annular wavefronts'', APPLIED OPTICS, 49, 33, 6489- 6501, 2010.

[9] V. Mahajan, and J. Díaz, ''Imaging characteristics of Zernike and annular polynomial aberrations'', APPLIED OPTICS, 52, 10, 2062- 2074, 2013.

[10] R. Noll, ''Zernike polynomials and atmospheric turbulence'', J. Opt. Soc. Am., 66, 3, 207-211, 1976.

[11] C. Zhao and J. Burge, ''Orthonormal vector polynomials in a unit circle, Part I: basis set derived from gradients of Zernike polynomials'', OPTICS EXPRESS, 15, 26, 18014-18024, 2007.

[12] C. Zhao and J. Burge, ''Orthonormal vector polynomials in a unit circle, Part II: completing the basis set'', OPTICS EXPRESS, 16, 9, 6586-6591, 2008.

[13] C. Zhao and J. Burge, ''Orthonormal Vector Polynomials in a unit Circle, Application: Fitting mapping distortions in a null test'', Proc of SPEI, 7426, 74260V, 2009.

[14] S. Roholah, B. Mehr, and M. Taeibi-Rahni " A Parallel implementation of Gram-Schmidt Algorithm for Dense Linear System of Equations" International Journal of Computer Applications 5,7, 0975 – 8887, 2010.

[15] G. Arfken, ''Mathematical Methods for Physicists'', 3rd.ed., chap.1, Miami university, Academic Press, Inc., USA, 1985.

Table (1): Orthonormal Zernike Annular Polynomials $A(\rho,\theta;\epsilon)$ in polar coordinates

1) 1

2) $\dfrac{2\rho\cos(\theta)}{\sqrt{\epsilon^2+1}}$

3) $\dfrac{2\rho\sin(\theta)}{\sqrt{\epsilon^2+1}}$

4) $\dfrac{\sqrt{3}(2\rho^2-1-\epsilon^2)}{(1-\epsilon^2)}$

5) $\dfrac{\sqrt{6}\rho^2\sin(2\theta)}{\sqrt{\epsilon^4+\epsilon^2+1}}$

6) $\dfrac{\sqrt{6}\rho^2\cos(2\theta)}{\sqrt{\epsilon^4+\epsilon^2+1}}$

7) $\dfrac{2\sqrt{2}\rho\sin(\theta)\left[3(\epsilon^2+1)\rho^2-2(\epsilon^4+\epsilon^2+1)\right]}{\left[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}\right]}$

8) $\dfrac{2\sqrt{2}\rho\cos(\theta)\left[\left[3(\epsilon^2+1)\rho^2-2(\epsilon^4+\epsilon^2+1)\right]\right]}{\left[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}\right]}$

9) $\dfrac{2\sqrt{2}\rho^3\sin(3\theta)}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$

10) $\dfrac{2\sqrt{2}\rho^3\cos(3\theta)}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$

11) $\dfrac{\sqrt{5}\left[6\rho^4-6(\epsilon^2+1)\rho^2+(\epsilon^4+4\epsilon^2+1)\right]}{(\epsilon^4-2\epsilon^2+1)}$

12) $\dfrac{\sqrt{10}\rho^2\cos(2\theta)[4(\epsilon^4+\epsilon^2+1)\rho^2-3(\epsilon^6+\epsilon^4+\epsilon^2+1)]}{[(\epsilon^2-1)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$

13) $\dfrac{\sqrt{10}\rho^2\sin(2\theta)\left[4(\epsilon^4+\epsilon^2+1)\rho^2-3(\epsilon^6+\epsilon^4+\epsilon^2+1)\right]}{[(\epsilon^2-1)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$

14) $\dfrac{\sqrt{10}\rho^4\cos(4\theta)}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

15) $\dfrac{\sqrt{10}\rho^4\sin(4\theta)}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

16) $\dfrac{2\sqrt{3}\rho\cos(\theta)\left[3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)+10(\epsilon^4+4\epsilon^2+1)\rho^4-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)\rho^2\right]}{\epsilon^4-2\epsilon^2+1} \times \sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$

17) $\dfrac{2\sqrt{3}\rho\sin(\theta)\left[3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)+10(\epsilon^4+4\epsilon^2+1)\rho^4-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)\rho^2\right]}{\epsilon^4-2\epsilon^2+1} \times \sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$

18) $\dfrac{2\sqrt{3}\rho^3\cos(3\theta)\left[5\rho^2(\epsilon^6+\epsilon^4+\epsilon^2+1)-4(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)\right]}{\epsilon^2-1} \times \sqrt{\epsilon^{18}+5\epsilon^{16}+15\epsilon^{14}+35\epsilon^{12}+44\epsilon^{10}+44\epsilon^8+35\epsilon^6+15\epsilon^4+5\epsilon^2+1}$

19) $\dfrac{2\sqrt{3}\rho^3\sin(3\theta)\left[(5\rho^2-4)+4(\epsilon^{10}-\epsilon^8)/(1-\epsilon^8)\right]}{(\epsilon^6+\epsilon^4+\epsilon^2+1)}$

20) $\dfrac{2\sqrt{3}\rho^5\cos(5\theta)}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

21) $\dfrac{2\sqrt{3}\rho^5\sin(5\theta)}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

Table (2): Zernike Annular Polynomials A(x,y;ϵ) in Cartesian coordinates

$$1)\ 1 \qquad 2)\ \frac{2x}{\sqrt{\epsilon^2+1}} \qquad 3)\ \frac{2y}{\sqrt{\epsilon^2+1}} \qquad 4)\ \frac{\sqrt{3}(2x^2+2y^2-1-\epsilon^2)}{(\epsilon^2-1)} \qquad 5)\ \frac{2\sqrt{6}xy}{\sqrt{\epsilon^4+\epsilon^2+1}} \qquad 6)\ \frac{\sqrt{6}(x^2-y^2)}{\sqrt{\epsilon^4+\epsilon^2+1}}$$

$$7)\ \frac{2\sqrt{2}[3(\epsilon^2+1)(x^2y+y^3)-2(\epsilon^4+\epsilon^2+1)y]}{[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}]} \qquad 8)\ \frac{2\sqrt{2}[3(\epsilon^2+1)(x^3+xy^2)-2(\epsilon^4+\epsilon^2+1)x]}{[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}]}$$

$$9)\ \frac{2\sqrt{2}(3yx^2-y^3)}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 10)\ \frac{2\sqrt{2}(x^3-3xy^2)}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 11)\ \frac{\sqrt{5}[6(x^4+2x^2y^2+y^4)-6(\epsilon^2+1)(x^2+y^2)+(\epsilon^4+4\epsilon^2+1)]}{(\epsilon^4-2\epsilon^2+1)}$$

$$12)\ \frac{\sqrt{10}[4(\epsilon^4+\epsilon^2+1)(x^4-y^4)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$$

$$13)\ \frac{2\sqrt{10}[4(\epsilon^4+\epsilon^2+1)(yx^3+xy^3)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)xy]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$$

$$14)\ \frac{\sqrt{10}(x^4-6x^2y^2+y^4)}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 15)\ \frac{4\sqrt{10}(yx^3-xy^3)}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

$$16)\ \left[\frac{2\sqrt{3}[3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)x+10(\epsilon^4+4\epsilon^2+1)(x^5+2x^3y^2+xy^4)-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)(x^3+xy^2)]}{\epsilon^4-2\epsilon^2+1}\right] \times\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$$

$$17)\ \left[\frac{2\sqrt{3}[3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)y+10(\epsilon^4+4\epsilon^2+1)(yx^4+2x^2y^3+y^5)-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)(yx^2+y^3)]}{\epsilon^4-2\epsilon^2+1}\right] \times\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$$

$$18)\ \frac{2\sqrt{3}[5(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^5-2x^3y^2-3y^4x)-4(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)(x^3-3xy^2)]}{\epsilon^2-1} \times\sqrt{(\epsilon^{18}+5\epsilon^{16}+15\epsilon^{14}+35\epsilon^{12}+44\epsilon^{10}+44\epsilon^8+35\epsilon^6+15\epsilon^4+5\epsilon^2+1)}$$

$$19)\ \frac{2\sqrt{3}[5(\epsilon^6+\epsilon^4+\epsilon^2+1)(3yx^4+2x^2y^3-y^5)-4(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)(3yx^2-y^3)]}{(\epsilon^6+\epsilon^4+\epsilon^2+1)}$$

$$20)\ \frac{2\sqrt{3}(x^5-10x^3y^2+5xy^4)}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 21)\ \frac{2\sqrt{3}(5yx^4-10x^2y^3+y^5)}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

Table (3): Annular ZPs A(x,y;ϵ) as function of circular ZPs, Z(x,y)

$$1)\ Z_1 \qquad 2)\ \frac{Z_2}{\sqrt{\epsilon^2+1}} \qquad 3)\ \frac{Z_3}{\sqrt{\epsilon^2+1}} \qquad 4)\ \frac{Z_4-\sqrt{3}Z_1\epsilon^2}{(1-\epsilon^2)} \qquad 5)\ \frac{Z_5}{\sqrt{\epsilon^4+\epsilon^2+1}} \qquad 6)\ \frac{Z_6}{\sqrt{\epsilon^4+\epsilon^2+1}}$$

$$7)\ \frac{[Z_7(\epsilon^2+1)-2\sqrt{2}Z_3\epsilon^4]}{[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}]} \qquad 8)\ \frac{[Z_8(\epsilon^2+1)-2\sqrt{2}Z_2\epsilon^4]}{[(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}]} \qquad 9)\ \frac{Z_9}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

$$10)\ \frac{Z_{10}}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 11)\ \frac{[Z_{11}-\sqrt{15}Z_4\epsilon^2+\sqrt{5}Z_1(\epsilon^4-\epsilon^2)]}{\sqrt{\epsilon^4-2\epsilon^2+1}}$$

$$12)\ \frac{[Z_{12}(\epsilon^4+\epsilon^2+1)-\sqrt{15}Z_6\epsilon^6]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$$

$$13)\ \frac{[Z_{13}(\epsilon^4+\epsilon^2+1)-\sqrt{15}Z_5\epsilon^6]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]} \qquad 14)\ \frac{Z14}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 15)\ \frac{Z15}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)}}$$

$$16)\ \frac{[Z_{16}(\epsilon^4+4\epsilon^2+1)-2\sqrt{6}\epsilon^4Z_8(\epsilon^2-3)+\sqrt{3}\epsilon^4Z_2(3\epsilon^4+4\epsilon^2+3)]\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}}{\epsilon^4-2\epsilon^2+1}$$

$$17)\ \frac{[Z_{17}(\epsilon^4+4\epsilon^2+1)-2\sqrt{6}\epsilon^4Z_7(\epsilon^2-3)+\sqrt{3}\epsilon^4Z_3(3\epsilon^4+4\epsilon^2+3)]\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}}{\epsilon^4-2\epsilon^2+1}$$

$$18)\ \frac{[Z_{18}(\epsilon^6+\epsilon^4+\epsilon^2+1)-2\sqrt{6}Z_{10}\epsilon^8]\sqrt{(\epsilon^{18}+5\epsilon^{16}+15\epsilon^{14}+35\epsilon^{12}+44\epsilon^{10}+44\epsilon^8+35\epsilon^6+15\epsilon^4+5\epsilon^2+1)}}{\epsilon^2-1}$$

$$19)\ \frac{[Z_{19}(\epsilon^6+\epsilon^4+\epsilon^2+1)-2\sqrt{6}Z_9\epsilon^8]}{(\epsilon^6+\epsilon^4+\epsilon^2+1)} \qquad 20)\ \frac{Z_{20}}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 21)\ \frac{Z_{21}}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

Table (4): orthonormal Gradients of Annular ZPs ∇A(x,y;ϵ) in Cartesian coordinate

$$1)\ 0 \qquad 2)\ \frac{2\hat{\imath}}{\sqrt{\epsilon^2+1}} \qquad 3)\ \frac{2\hat{\jmath}}{\sqrt{\epsilon^2+1}} \qquad 4)\ \frac{4\sqrt{3}(x\hat{\imath}+y\hat{\jmath})}{(1-\epsilon^2)} \qquad 5)\ \frac{2\sqrt{6}(y\hat{\imath}+x\hat{\jmath})}{\sqrt{\epsilon^4+\epsilon^2+1}} \qquad 6)\ \frac{2\sqrt{6}(x\hat{\imath}-y\hat{\jmath})}{\sqrt{\epsilon^4+\epsilon^2+1}}$$

$$7)\ \frac{2\sqrt{2}\left[6(\epsilon^2+1)xy\hat{\imath}+\left(3(\epsilon^2+1)(x^2+3y^2)-2(\epsilon^4+\epsilon^2+1)\right)\hat{\jmath}\right]}{(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}}$$

$$8)\ \frac{2\sqrt{2}\left[\left(3(\epsilon^2+1)(3x^2+y^2)-2(\epsilon^4+\epsilon^2+1)\right)\hat{\imath}+6(\epsilon^2+1)xy\hat{\jmath}\right]}{(1-\epsilon^2)\sqrt{\epsilon^6+5\epsilon^4+5\epsilon^2+1}} \qquad 9)\ \frac{6\sqrt{2}[2xy\hat{\imath}+(x^2-y^2)\hat{\jmath}]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

$$10)\ \frac{6\sqrt{2}[(x^2-y^2)\hat{\imath}-2xy\hat{\jmath}]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 11)\ \frac{12\sqrt{5}[(2(x^3+xy^2)-(\epsilon^2+1)x)\hat{\imath}+(2(y^3+yx^2)-(\epsilon^2+1)y)\hat{\jmath}]}{\epsilon^4-2\epsilon^2+1}$$

$$12)\ \frac{2\sqrt{10}[(8(\epsilon^4+4\epsilon^2+1)x^3-3(\epsilon^6+\epsilon^4+\epsilon^2+1)x)\hat{\imath}-(8(\epsilon^4+4\epsilon^2+1)y^3-3(\epsilon^6+\epsilon^4+\epsilon^2+1)y)\hat{\jmath}]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$$

$$13)\ \frac{2\sqrt{10}[(4(\epsilon^4+\epsilon^2+1)(3yx^2+y^3)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)y)\hat{\imath}+(4(\epsilon^4+\epsilon^2+1)(x^3+3xy^2)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)x)\hat{\jmath}]}{[(1-\epsilon^2)\sqrt{\epsilon^{12}+5\epsilon^{10}+15\epsilon^8+18\epsilon^6+15\epsilon^4+5\epsilon^2+1}]}$$

$$14)\ \frac{4\sqrt{10}[(x^3-3xy^2)\hat{\imath}+(y^3-3yx^2)\hat{\jmath}]}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 15)\ \frac{4\sqrt{10}[(3yx^2-y^3)\hat{\imath}+(x^3-3xy^2)\hat{\jmath}]}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

$$16)\ 2\sqrt{3}\left[\frac{\left[\left(3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)+10(\epsilon^4+4\epsilon^2+1)(5x^4+6x^2y^2+y^4)-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)(3x^2+y^2)\right)\hat{\imath}\right]}{\epsilon^4-2\epsilon^2+1}+\frac{[(40(\epsilon^4+4\epsilon^2+1)(yx^3+xy^3)-24(\epsilon^6+4\epsilon^4+4\epsilon^2+1)xy)\hat{\jmath}]}{\epsilon^4-2\epsilon^2+1}\right]\times\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$$

$$17)\ 2\sqrt{3}\left[\frac{[(40(\epsilon^4+4\epsilon^2+1)(yx^3+xy^3)-24(\epsilon^6+4\epsilon^4+4\epsilon^2+1)xy)\hat{\imath}]}{\epsilon^4-2\epsilon^2+1}+\frac{\left[\left(3(\epsilon^8+4\epsilon^6+10\epsilon^4+4\epsilon^2+1)+10(\epsilon^4+4\epsilon^2+1)(x^4+6x^2y^2+5y^4)-12(\epsilon^6+4\epsilon^4+4\epsilon^2+1)(x^2+3y^2)\right)\hat{\jmath}\right]}{\epsilon^4-2\epsilon^2+1}\right]\times\sqrt{\epsilon^{10}+13\epsilon^8+46\epsilon^6+46\epsilon^4+13\epsilon^2+1}$$

$$18)\ 2\sqrt{3}\left[\frac{\left[\left(5(\epsilon^6+\epsilon^4+\epsilon^2+1)(5x^4-6x^2y^2-3y^4)-12(\epsilon^4+\epsilon^2+1)(x^2-y^2)\right)\hat{\imath}\right]M_1}{\epsilon^2-1}+\frac{[-(20(\epsilon^6+\epsilon^4+\epsilon^2+1)(yx^3+3xy^3)-24(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)xy)\hat{\jmath}]M_1}{\epsilon^2-1}\right]$$

$$19)\ 2\sqrt{3}\left[\frac{[(20(\epsilon^6+\epsilon^4+\epsilon^2+1)(3yx^3+xy^3)-24(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)xy)\hat{\imath}]}{\epsilon^6+\epsilon^4+\epsilon^2+1}+\frac{\left[\left(5(\epsilon^6+\epsilon^4+\epsilon^2+1)(3x^4+6x^2y^2-5y^4)-12(\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)\right)\hat{\jmath}\right]}{\epsilon^6+\epsilon^4+\epsilon^2+1}\right.$$

$$20)\ \frac{10\sqrt{3}[(x^4-6x^2y^2+y^4)\hat{\imath}-4(yx^3-xy^3)\hat{\jmath}]}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}} \qquad 21)\ \frac{10\sqrt{3}[4(yx^3-xy^3)\hat{\imath}+(x^4-6x^2y^2+y^4)\hat{\jmath}]}{\sqrt{\epsilon^{10}+\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$$

Table (5): Orthonormal Vector Annular ZP ($\vec{S}$) in Cartesian coordinates

1) 0 2) $\hat{\imath}$ 3) $\hat{\jmath}$ 4) $\frac{\sqrt{2}(x\,\hat{\imath}+y\,\hat{\jmath})}{\sqrt{\epsilon^2+1}}$ 5) $\frac{\sqrt{2}(y\,\hat{\imath}+x\,\hat{\jmath})}{\sqrt{\epsilon^2+1}}$ 6) $\frac{\sqrt{2}(x\,\hat{\imath}-y\,\hat{\jmath})}{\sqrt{\epsilon^2+1}}$

7) $\frac{\sqrt{3}\left[2xy\,\hat{\imath}+\left((x^2+3y^2)-(\epsilon^2+1)\right)\hat{\jmath}\right]}{\sqrt{2\epsilon^4-\epsilon^2+2}}$ 8) $\frac{\sqrt{3}\left[\left((3x^2+y^2)-(\epsilon^2+1)\right)\hat{\imath}+2xy\,\hat{\jmath}\right]}{\sqrt{2\epsilon^4-\epsilon^2+2}}$

9) $\frac{\sqrt{3}\left[2xy\,\hat{\imath}+(x^2-y^2)\hat{\jmath}\right]}{\sqrt{\epsilon^4+\epsilon^2+1}}$ 10) $\frac{\sqrt{3}\left[(x^2-y^2)\hat{\imath}-2xy\,\hat{\jmath}\right]}{\sqrt{\epsilon^4+\epsilon^2+1}}$

11) $\frac{2\left[\left(3(\epsilon^2+1)(x^3+xy^2)-2(\epsilon^4+\epsilon^2+1)x\right)\hat{\imath}+\left(3(\epsilon^2+1)(yx^2+y^3)-2(\epsilon^4+\epsilon^2+1)y\right)\hat{\jmath}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8-4\epsilon^6-4\epsilon^4+3\epsilon^2+1}}$

12) $\frac{2\sqrt{2}\left[\left(2x^3(\epsilon^2+1)-(\epsilon^4+\epsilon^2+1)x\right)\hat{\imath}+\left((\epsilon^4+\epsilon^2+1)y-2y^3(\epsilon^2+1)\right)\hat{\jmath}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

13) $\frac{2\sqrt{2}\left[\left((\epsilon^2+1)(3yx^2+y^3)-(\epsilon^4+\epsilon^2+1)y\right)\hat{\imath}+\left((\epsilon^2+1)(3xy^2+x^3)-(\epsilon^4+\epsilon^2+1)x\right)\hat{\jmath}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

14) $\frac{2\left[(x^3-3xy^2)\hat{\imath}-(3yx^2-y^3)\hat{\jmath}\right]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$ 15) $\frac{2\left[(3yx^2-y^3)\hat{\imath}+(x^3-3xy^2)\hat{\jmath}\right]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$

16) $2\sqrt{5}\left[\frac{\left[\left((\epsilon^8+2\epsilon^6-3\epsilon^4+2\epsilon^2+1)+(2\epsilon^4-\epsilon^2+2)(5x^4+6y^2x^2+y^4)-3(\epsilon^6+1)(3x^2+y^2)\right)\hat{\imath}\right]}{M_2^2M_3}+\frac{\left[\left(4(2\epsilon^4-\epsilon^2+2)(yx^3+xy^3)-6(\epsilon^6+1)yx\right)\hat{\jmath}\right]}{M_2^2M_3}\right]$

17) $2\sqrt{5}\left[\frac{\left[4(2\epsilon^4-\epsilon^2+2)(yx^3+xy^3)-6(\epsilon^6+1)yx\;\hat{\imath}\right]}{M_2^2M_3}+\frac{\left[\left((\epsilon^8+2\epsilon^6-3\epsilon^4+2\epsilon^2+1)+(2\epsilon^4-\epsilon^2+2)(x^4+6y^2x^2+5y^4)-3(\epsilon^6+1)(x^2+3y^2)\right)\hat{\jmath}\right]}{M_2^2M_3}\right]$

18) $\sqrt{5}\left[\frac{\left[\left((\epsilon^4+\epsilon^2+1)(5x^4-6x^2y^2-3y^4)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)\right)\hat{\imath}\right]}{\sqrt{2\epsilon^{16}+6\epsilon^{14}+12\epsilon^{12}+\epsilon^{10}+3\epsilon^8+\epsilon^6+12\epsilon^4+6\epsilon^2+2}}+\frac{\left[\left(6(\epsilon^6+\epsilon^4+\epsilon^2+1)xy-4(\epsilon^4+\epsilon^2+1)(yx^3+3xy^3)\right)\hat{\jmath}\right]}{\sqrt{2\epsilon^{16}+6\epsilon^{14}+12\epsilon^{12}+\epsilon^{10}+3\epsilon^8+\epsilon^6+12\epsilon^4+6\epsilon^2+2}}\right]$

19) $\sqrt{5}\left[\frac{\left[\left(4(\epsilon^4+\epsilon^2+1)(3yx^3+xy^3)-6(\epsilon^6+\epsilon^4+\epsilon^2+1)xy\right)\hat{\imath}\right]}{\sqrt{(2\epsilon^{16}+6\epsilon^{14}+12\epsilon^{12}+\epsilon^{10}+3\epsilon^8+\epsilon^6+12\epsilon^4+6\epsilon^2+2)}}+\frac{\left[\left((\epsilon^4+\epsilon^2+1)(3x^4+6x^2y^2-5y^4)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)\right)\hat{\jmath}\right]}{\sqrt{(2\epsilon^{16}+6\epsilon^{14}+12\epsilon^{12}+\epsilon^{10}+3\epsilon^8+\epsilon^6+12\epsilon^4+6\epsilon^2+2)}}\right]$

20) $\frac{\sqrt{5}\left[(x^4-6x^2y^2+y^4)\hat{\imath}-4(yx^3-xy^3)\hat{\jmath}\right]}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$ 21) $\frac{\sqrt{5}\left[4(yx^3-xy^3)\hat{\imath}+(x^4-6x^2y^2+y^4)\hat{\jmath}\right]}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

Table (6): Orthonormal vector annular ZP ($\vec{S}$) as a function of circular ZP Z(x,y)

1) 0 2) $Z_1\,\hat{\imath}$ 3) $Z_1\,\hat{\jmath}$ 4) $\frac{Z_2\,\hat{\imath}+Z_3\,\hat{\jmath}}{\sqrt{2\epsilon^2+2}}$ 5) $\frac{Z_3\,\hat{\imath}+Z_2\,\hat{\jmath}}{\sqrt{2\epsilon^2+2}}$ 6) $\frac{Z_2\,\hat{\imath}-Z_3\,\hat{\jmath}}{\sqrt{2\epsilon^2+2}}$

7) $\frac{2\sqrt{3}Z_5\,\hat{\imath}+2\left(-\sqrt{3}Z_6+\sqrt{6}Z_4-3\sqrt{2}\epsilon^2Z_1\right)\hat{\jmath}}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$ 8) $\frac{2\left(-\sqrt{3}Z_6+\sqrt{6}Z_4-3\sqrt{2}\epsilon^2Z_1\right)\hat{\imath}+2\sqrt{3}Z_5\,\hat{\jmath}}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$

9) $\frac{Z_5\,\hat{\imath}+Z_6\,\hat{\jmath}}{\sqrt{2\epsilon^4+2\epsilon^2+2}}$ 10) $\frac{Z_6\,\hat{\imath}-Z_5\,\hat{\jmath}}{\sqrt{2\epsilon^4+2\epsilon^2+2}}$ 11) $\frac{\left[\left(Z_8(\epsilon^2+1)-2\sqrt{2}\epsilon^4Z_2\right)\hat{\imath}+\left(Z_7(\epsilon^2+1)-2\sqrt{2}\epsilon^4Z_3\right)\hat{\jmath}\right]}{\sqrt{2\epsilon^{10}+6\epsilon^8-8\epsilon^6-8\epsilon^4+6\epsilon^2+2}}$

12) $\frac{\left[\left(Z_{10}(\epsilon^2+1)/2+Z_8(\epsilon^2+1)/2-\sqrt{2}Z_2\epsilon^4\right)\hat{\imath}+\left(Z_9(\epsilon^2+1)/2-Z_7(\epsilon^2+1)/2+\sqrt{2}Z_3\epsilon^4\right)\hat{\jmath}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

13) $\frac{\left[\left(Z_9(\epsilon^2+1)/2+Z_7(\epsilon^2+1)/2-\sqrt{2}Z_3\epsilon^4\right)\hat{\imath}+\left(-Z_{10}(\epsilon^2+1)/2+Z_8(\epsilon^2+1)/2-\sqrt{2}Z_2\epsilon^4\right)\hat{\jmath}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

14) $\frac{[Z_{10}\,\hat{\imath}-Z_9\,\hat{\jmath}]}{\sqrt{2\epsilon^6+2\epsilon^4+2\epsilon^2+2}}$ 15) $\frac{[Z_9\,\hat{\imath}+Z_{10}\,\hat{\jmath}]}{\sqrt{2\epsilon^6+2\epsilon^4+2\epsilon^2+2}}$

16) $\frac{\left[\left(Z_{12}M_{13}^2+\sqrt{2}Z_{11}M_{13}^2-\sqrt{15}Z_6(2\epsilon^6-2\epsilon^4+\epsilon^2)-\sqrt{30}Z_4(2\epsilon^6-2\epsilon^4+\epsilon^2)+2\sqrt{10}\,Z_1(\epsilon^8-\epsilon^6-\epsilon^4+\epsilon^2)\right)\hat{\imath}\right]}{\sqrt{2}M_2^2M_3}+\frac{\left[\left(Z_{13}M_{13}^2-\sqrt{15}\,Z_5(2\epsilon^6-2\epsilon^4+\epsilon^2)\right)\hat{\jmath}\right]}{\sqrt{2}M_2^2M_3}$

17) $\frac{\left[\left(Z_{13}M_{13}^2-\sqrt{15}\,Z_5(2\epsilon^6-2\epsilon^4+\epsilon^2)\right)\hat{\imath}\right]}{\sqrt{2}M_2^2M_3}+\frac{\left[\left(-Z_{12}M_{13}^2+\sqrt{2}Z_{11}M_{13}^2+\sqrt{15}Z_6(2\epsilon^6-2\epsilon^4+\epsilon^2)-\sqrt{30}Z_4(2\epsilon^6-2\epsilon^4+\epsilon^2)+2\sqrt{10}\,Z_1(\epsilon^8-\epsilon^6-\epsilon^4+\epsilon^2)\right)\hat{\jmath}\right]}{\sqrt{2}M_2^2M_3}$

18) $\frac{\left[\left(Z_{14}(\epsilon^4+\epsilon^2+1)+Z_{12}(\epsilon^4+\epsilon^2+1)-\sqrt{15}\,\epsilon^6Z_6\right)\hat{\imath}+\left(Z_{15}(\epsilon^4+\epsilon^2+1)-Z_{13}(\epsilon^4+\epsilon^2+1)+\sqrt{15}\,\epsilon^6Z_5\right)\hat{\jmath}\right]}{/\sqrt{2}\,M_4}$

19) $\frac{\left[\left(Z_{15}(\epsilon^4+\epsilon^2+1)+Z_{13}(\epsilon^4+\epsilon^2+1)-\sqrt{15}\,\epsilon^6Z_5\right)\hat{\imath}+\left(-Z_{14}(\epsilon^4+\epsilon^2+1)+Z_{12}(\epsilon^4+\epsilon^2+1)-\sqrt{15}\,\epsilon^6Z_6\right)\hat{\jmath}\right]}{/\sqrt{2}\,M_4}$

20) $\frac{[Z_{14}\,\hat{\imath}-Z_{15}\,\hat{\jmath}]}{\sqrt{2\epsilon^8+2\epsilon^6+2\epsilon^4+2\epsilon^2+2}}$ 21) $\frac{[Z_{15}\,\hat{\imath}+Z_{14}\,\hat{\jmath}]}{\sqrt{2\epsilon^8+2\epsilon^6+2\epsilon^4+2\epsilon^2+2}}$

Table (7): Orthonormal vector annular ZP ($\vec{S}$) as a function of gradient of circular ZP ∇Z(x,y)

1) 0 2) $\frac{\nabla Z_2}{\sqrt{4}}$ 3) $\frac{\nabla Z_3}{\sqrt{4}}$ 4) $\frac{\nabla Z_4}{\sqrt{24\epsilon^2+24}}$ 5) $\frac{\nabla Z_5}{\sqrt{12\epsilon^2+12}}$ 6) $\frac{\nabla Z_6}{\sqrt{12\epsilon^2+12}}$

7) $\frac{\left[\nabla Z_7-\sqrt{2}(3\epsilon^2+1)\nabla Z_3\right]}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$ 8) $\frac{\left[\nabla Z_8-\sqrt{2}(3\epsilon^2+1)\nabla Z_2\right]}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$ 9) $\frac{\nabla Z_9}{\sqrt{24\epsilon^4+24\epsilon^2+24}}$ 10) $\frac{\nabla Z_{10}}{\sqrt{24\epsilon^4+24\epsilon^2+24}}$

11) $\frac{\left[\nabla Z_{11}(\epsilon^2+1)-\sqrt{\frac{5}{3}}(4\epsilon^4+\epsilon^2+1)\nabla Z_4\right]}{\sqrt{80\epsilon^{10}+240\epsilon^8+240\epsilon^2+80}}$ 12) $\frac{\left[\nabla Z_{12}(\epsilon^2+1)-\sqrt{\frac{5}{3}}(4\epsilon^4+\epsilon^2+1)\nabla Z_6\right]}{\sqrt{80\epsilon^{10}+240\epsilon^8+240\epsilon^2+80}}$

13) $\frac{\left[\nabla Z_{13}(\epsilon^2+1)-\sqrt{\frac{5}{3}}(4\epsilon^4+\epsilon^2+1)\nabla Z_5\right]}{\sqrt{80\epsilon^{10}+240\epsilon^8+240\epsilon^2+80}}$ 14) $\frac{\nabla Z_{14}}{\sqrt{40\epsilon^6+40\epsilon^4+40\epsilon^2+40}}$ 15) $\frac{\nabla Z_{15}}{\sqrt{40\epsilon^6+40\epsilon^4+40\epsilon^2+40}}$

16) $\frac{\left[\nabla Z_{16}(2\epsilon^4-\epsilon^2+2)-\sqrt{\frac{3}{2}}(10\epsilon^6+2\epsilon^4-\epsilon^2+12)\nabla Z_8\right.}{\sqrt{60}\,M_2^2M_3}$

17) $\frac{[\nabla Z_{17}(2\epsilon^4-\epsilon^2+2)-\sqrt{\frac{3}{2}}(10\epsilon^6+2\epsilon^4-\epsilon^2+12)\nabla Z_7}{\sqrt{60}\, M_2^2 M_3}$

18) $\left[\nabla Z_{18}(\epsilon^4+\epsilon^2+1)-\sqrt{\frac{3}{2}}(5\epsilon^6+\epsilon^4+\epsilon^2+1)\nabla Z_{10}\right]/\sqrt{60}M_4$

19) $\left[\nabla Z_{19}(\epsilon^4+\epsilon^2+1)-\sqrt{\frac{3}{2}}(5\epsilon^6+\epsilon^4+\epsilon^2+1)\nabla Z_9\right]/\sqrt{60}M_4$

20) $\frac{\nabla Z_{20}}{\sqrt{60\epsilon^8+60\epsilon^6+60\epsilon^4+60\epsilon^2+60}}$ 21) $\frac{\nabla Z_{21}}{\sqrt{60\epsilon^8+60\epsilon^6+60\epsilon^4+60\epsilon^2+60}}$

Table (8): The normalization constants for orthogonal vector annular ZP.

1) 0 2) 1/2 n 3) 1/2 4) $\frac{1}{\sqrt{24\epsilon^2+24}}$ 5) $\frac{1}{\sqrt{12\epsilon^2+12}}$ 6) $\frac{1}{\sqrt{12\epsilon^2+12}}$

7) $\frac{1}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$ 8) $\frac{1}{\sqrt{48\epsilon^4-24\epsilon^2+48}}$ 9) $\frac{1}{\sqrt{24\epsilon^4+24\epsilon^2+24}}$ 10) $\frac{1}{\sqrt{24\epsilon^4+24\epsilon^2+24}}$

11) $\frac{(\epsilon^4-1)/\sqrt{80}}{\sqrt{\epsilon^{10}+\epsilon^8-8\epsilon^6+8\epsilon^4-\epsilon^2-1}}$ 12) $\frac{(\epsilon^4-1)/\sqrt{80}}{\sqrt{\epsilon^{10}+\epsilon^8-4\epsilon^6+4\epsilon^4-\epsilon^2-1}}$ 13) $\frac{(\epsilon^4-1)/\sqrt{80}}{\sqrt{\epsilon^{10}+\epsilon^8-4\epsilon^6+4\epsilon^4-\epsilon^2-1}}$

14) $\frac{1}{\sqrt{40\epsilon^6+40\epsilon^4+40\epsilon^2+40}}$ 15) $\frac{1}{\sqrt{40\epsilon^6+40\epsilon^4+40\epsilon^2+40}}$

16) $\frac{(2\epsilon^6-3\epsilon^4+3\epsilon^2-2)/\sqrt{240}}{\sqrt{\epsilon^{14}-3\epsilon^{12}+11\epsilon^{10}-25\epsilon^8+25\epsilon^6-11\epsilon^4+3\epsilon^2-1}}$ 17) $\frac{(2\epsilon^6-3\epsilon^4+3\epsilon^2-2)/\sqrt{240}}{\sqrt{\epsilon^{14}-3\epsilon^{12}+11\epsilon^{10}-25\epsilon^8+25\epsilon^6-11\epsilon^4+3\epsilon^2-1}}$

18) $\frac{(\epsilon^6+2\epsilon^4+2\epsilon^2+1)/\sqrt{60}}{\sqrt{2\epsilon^{14}+2\epsilon^{12}+2\epsilon^{10}-15\epsilon^8+15\epsilon^6-2\epsilon^4-2\epsilon^2-2}}$ 19) $\frac{(\epsilon^6+2\epsilon^4+2\epsilon^2+1)/\sqrt{60}}{\sqrt{2\epsilon^{14}+2\epsilon^{12}+2\epsilon^{10}-15\epsilon^8+15\epsilon^6-2\epsilon^4-2\epsilon^2-2}}$

20) $\frac{1}{\sqrt{60\epsilon^8+60\epsilon^6+60\epsilon^4+60\epsilon^2+60}}$ 21) $\frac{1}{\sqrt{60\epsilon^8+60\epsilon^6+60\epsilon^4+60\epsilon^2+60}}$

Table (9): Orthonormal vector annular ZP ($\vec{T}$) in Cartesian coordinates

1) 0 2) $-\hat{j}$ 3) $\hat{i}$ 4) $\frac{\sqrt{2}(y\,\hat{i}-x\,\hat{j})}{\sqrt{\epsilon^2+1}}$ 5) $\frac{\sqrt{2}(x\,\hat{i}-y\,\hat{j})}{\sqrt{\epsilon^2+1}}$ 6) $\frac{\sqrt{2}(-y\,\hat{i}-x\,\hat{j})}{\sqrt{\epsilon^2+1}}$

7) $\frac{\sqrt{3}\left[\left((x^2+3y^2)-(\epsilon^2+1)\right)\hat{i}-2xy\,\hat{j}\right]}{\sqrt{2\epsilon^4-\epsilon^2+2}}$ 8) $\frac{\sqrt{3}\left[2xy\,\hat{i}+\left((\epsilon^2+1)-(3x^2+y^2)\right)\hat{j}\right]}{\sqrt{2\epsilon^4-\epsilon^2+2}}$ 9) $\frac{\sqrt{3}\left[(x^2-y^2)\hat{i}-2xy\,\hat{j}\right]}{\sqrt{\epsilon^4+\epsilon^2+1}}$

10) $\frac{\sqrt{3}\left[-2xy\,\hat{i}-(x^2-y^2)\hat{j}\right]}{\sqrt{\epsilon^4+\epsilon^2+1}}$ 11) $\frac{\left[(3(\epsilon^2+1)(yx^2+y^3)-2(\epsilon^4+\epsilon^2+1)y)\hat{i}-(3(\epsilon^2+1)(x^3+xy^2)-2(\epsilon^4+\epsilon^2+1)x)\hat{j}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8-4\epsilon^6-4\epsilon^4+3\epsilon^2+1}}$

12) $\frac{2\sqrt{2}\left[((\epsilon^4+\epsilon^2+1)y-2(\epsilon^2+1)y^3)\hat{i}+((\epsilon^4+\epsilon^2+1)x-2(\epsilon^2+1)x^3)\hat{j}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

13) $\frac{2\sqrt{2}\left[\left((\epsilon^2+1)(x^3+3xy^2)-(\epsilon^4+\epsilon^2+1)x\right)\hat{i}-\left((\epsilon^2+1)(3yx^2+y^3)-(\epsilon^4+\epsilon^2+1)y\right)\hat{j}\right]}{\sqrt{\epsilon^{10}+3\epsilon^8+3\epsilon^2+1}}$

14) $\frac{2\left[(y^3-3yx^2)\hat{i}-(x^3-3xy^2)\hat{j}\right]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$ 15) $\frac{2\left[(x^3-3xy^2)\hat{i}-(3yx^2-y^3)\hat{j}\right]}{\sqrt{\epsilon^6+\epsilon^4+\epsilon^2+1}}$

16) $2\sqrt{5}\left[\frac{4(2\epsilon^4-\epsilon^2+2)(yx^3+xy^3)-6(\epsilon^6+1)yx)}{M_2^2M_3}\hat{i}\right]-$

$2\sqrt{5}\left[\frac{\left((\epsilon^8+2\epsilon^6-3\epsilon^4+2\epsilon^2+1)+(2\epsilon^4-\epsilon^2+2)(5x^4+6y^2x^2+y^4)-3(\epsilon^6+1)(3x^2+y^2)\right)}{M_2^2M_3}\hat{j}\right]$

17) $2\sqrt{5}\left[\frac{\left((\epsilon^8+2\epsilon^6-3\epsilon^4+2\epsilon^2+1)+(2\epsilon^4-\epsilon^2+2)(x^4+6y^2x^2+5y^4)-3(\epsilon^6+1)(x^2+3y^2)\right)}{M_2^2M_3}\hat{i}\right]-$

$2\sqrt{5}\left[\frac{(4(2\epsilon^4-\epsilon^2+2)(yx^3+xy^3)-6(\epsilon^6+1)yx)}{M_2^2M_3}\hat{j}\right]$

18) $\sqrt{5}\left[\frac{\left(6(\epsilon^6+\epsilon^4+\epsilon^2+1)xy-4(\epsilon^4+\epsilon^2+1)(yx^3+3xy^3)\right)\hat{i}}{M_4}\right]$

$+\sqrt{5}\left[\frac{\left(3(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)-(\epsilon^4+\epsilon^2+1)(5x^4-6x^2y^2-3y^4)\right)\hat{j}}{M_4}\right]$

19) $\sqrt{5}\left[\frac{\left((\epsilon^4+\epsilon^2+1)(3x^4+6x^2y^2-5y^4)-3(\epsilon^6+\epsilon^4+\epsilon^2+1)(x^2-y^2)\right)\hat{i}}{M_4}\right]$

$+\sqrt{5}\left[\frac{\left(6(\epsilon^6+\epsilon^4+\epsilon^2+1)xy-4(\epsilon^4+\epsilon^2+1)(3yx^3+xy^3)\right)\hat{j}}{M_4}\right]$

20) $\frac{\sqrt{5}[-4(xy^3-yx^3)\,\hat{i}-(x^4-6x^2y^2+y^4)\,\hat{j}]}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$ 21) $\frac{\sqrt{5}[(x^4-6x^2y^2+y^4)\,\hat{i}-4(yx^3}{\sqrt{\epsilon^8+\epsilon^6+\epsilon^4+\epsilon^2+1}}$

where:

$$M_1=(\epsilon^{18}+5\epsilon^{16}+15\epsilon^{14}+35\epsilon^{12}+44\epsilon^{10}+44\epsilon^8+35\epsilon^6+15\epsilon^4+5\epsilon^2+1)^{1/2}$$

$$M_2^2=(2\epsilon^4-\epsilon^2+2)$$

$$M_3=(2\epsilon^{15}-2\epsilon^{14}-3\epsilon^{13}+3\epsilon^{12}+19\epsilon^{11}-19\epsilon^{10}-26\epsilon^9+26\epsilon^8+26\epsilon^7-26\epsilon^6-19\epsilon^5+19\epsilon^4+3\epsilon^3-3\epsilon^2-2\epsilon+2)^{1/2}$$

$$M_4=(2\epsilon^{16}+6\epsilon^{14}+12\epsilon^{12}+\epsilon^{10}+3\epsilon^8+\epsilon^6+12\epsilon^4+6\epsilon^2+2)^{1/2}$$

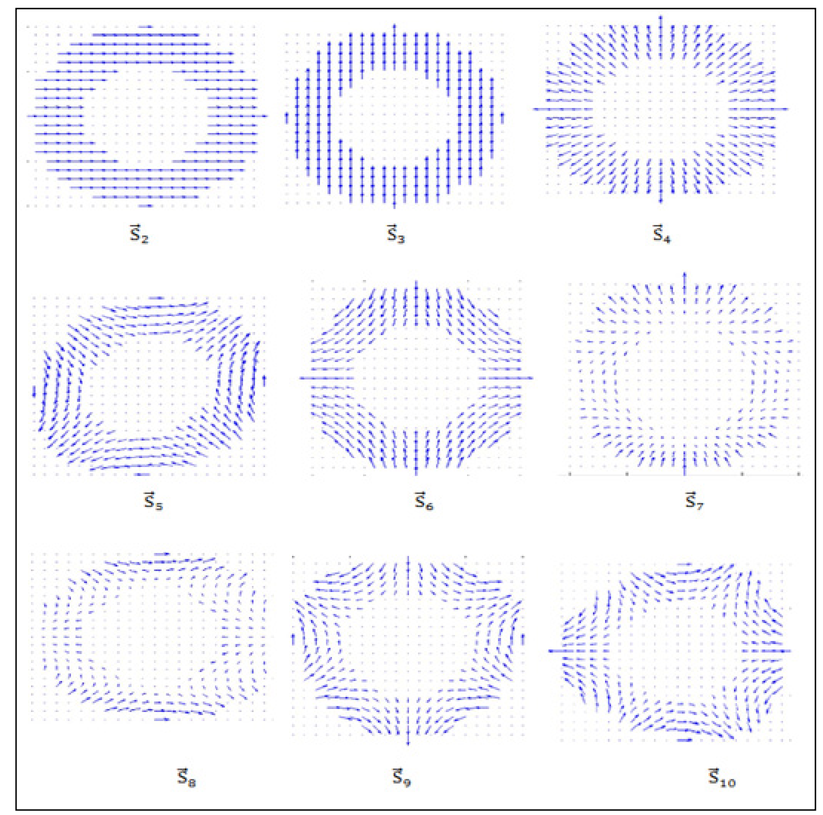

Fig. (1): Plots of first (9) orthonormal annular vector ZPs', ($\vec{S}$) , with (ϵ=0.2)

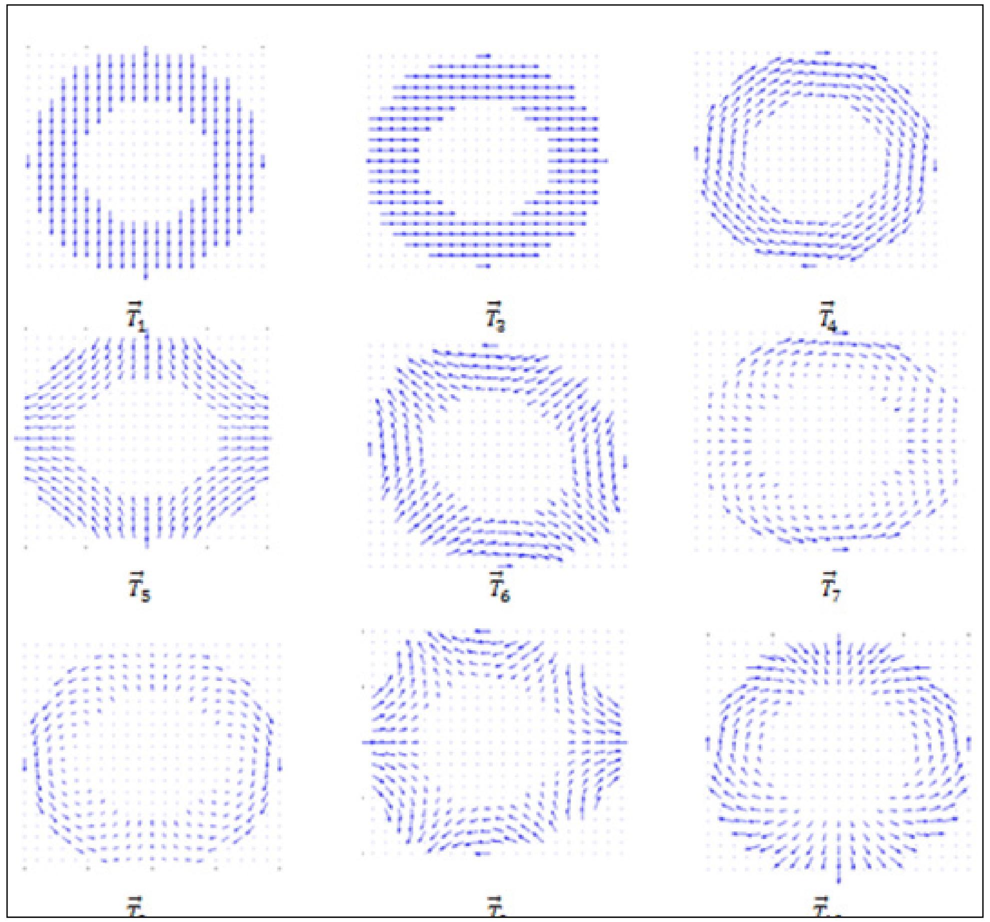

Fig. (2): Plots of first (9) orthonormal annular vector ZP.s', ($\vec{T}$), with (ϵ=0.2)

Antibiotics Susceptibility of Bacteria Aeromonas Hydrophila

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Received Date: 8 / 10 / 2015
Accepted Date: 19 / 1 / 2016

الخلاصة

من مجموع (294) عينة جمعت من مرضى أطفال بأعمار من شهر واحد الى (6) سنوات ممن يعانون الاسهال للفترة من كانون الثاني 2012– شباط (2013) في مدينة السماوة – العراق. شخصت العينات المشكوك فيها بالطرق المختبرية التقليدية وبتقنية تفاعل تسلسل البلمرة RNA PCR (16S) اظهرت النتائج (12) عزلة موجبة لبكتريا Aeromonas hydrophila وقد شخصت مجموعة من عوامل الضراوة للبكتريا واظهرت جميع العزلات قدرتها على انتاج الهيمولايسين والبروتيز واللايبيز والفوسفولايبيز وامتلاكها المحفظة وقدرتها على الحركة. كما اظهرت هذه العزلات مقاومتها لبعض المضادات الحيوية مثل الاموكسيلين وامبيسيلين وسيفالوسبورين وسيفوتاكسيم بينما كل العزلات كانت حساسة لكل من حامض الناليديليسك والتراسايكلين وجنتامايسين.

الكلمات المفتاحية

Aanomores hpordyhali تشخيص بعض عوامل الضراوة وتاثير بعض المضادات الحياتية.